# Order from Disorder: Chaos, Turbulence and Recurrent flow

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## Introduction

The evolution of a fluid through its lifetime can be complex and chaotic, and whilst the consequences that these systems have on our lives are numerous and significant, we still do not fully understand all aspects of its behaviour.

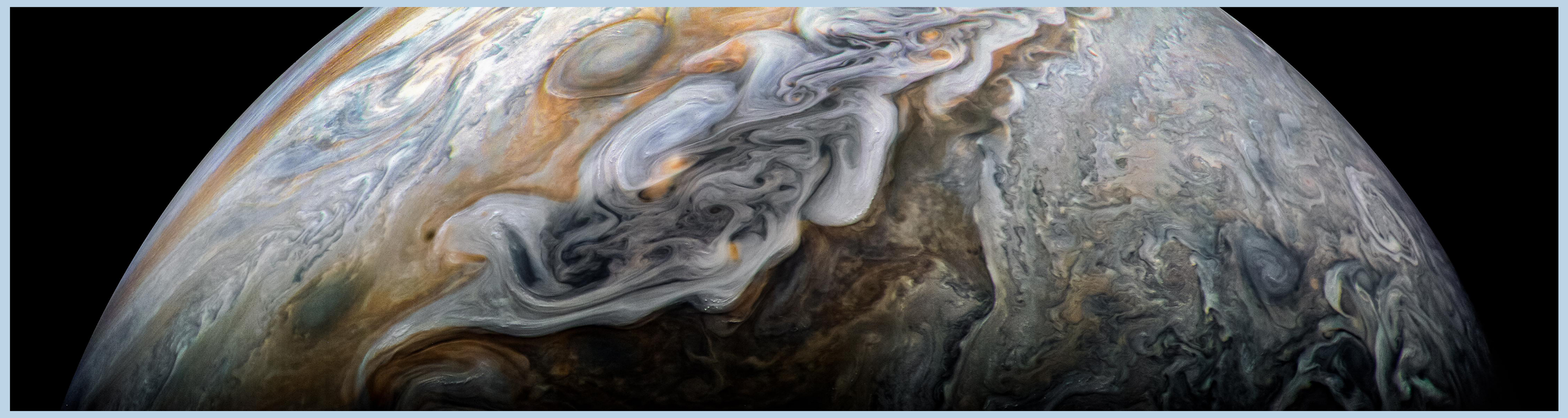

**Figure 1:** An illustration of large scale turbulence in Jupiter's atmosphere.

The chaotic trajectory of a turbulent flow through phase space, Figure (2), has been shown to shadow unstable periodic orbits, whose unstable nature means that the solution will not follow them exactly. We seek to improve the current method for finding these orbits by adding direction data from the phase space into the residual (1).

## Recurrent flow analysis

We must first consider a turbulent flow as a dynamical system whose properties evolve over time. To do this we simulate a fluid flow using numerical techniques and record data from this. Recurrent flow analysis exploits the large quantity of data collected by comparing the state of the system at different times and computing a residual $R(t,T)$ which we seek to minimise in the search for periodic orbits.

$$R(t,T) = \min_{0\leq s<2\pi} \min_{m\in 0,1,\ldots,n-1} \frac{\sum_j \sum_l |\Omega_{jl}(t)e^{i\alpha js+2iml\pi/n} - \Omega_{jl}(t-T)|^2}{\sum_j \sum_l |\Omega_{jl}(t)|^2} \qquad (1)$$

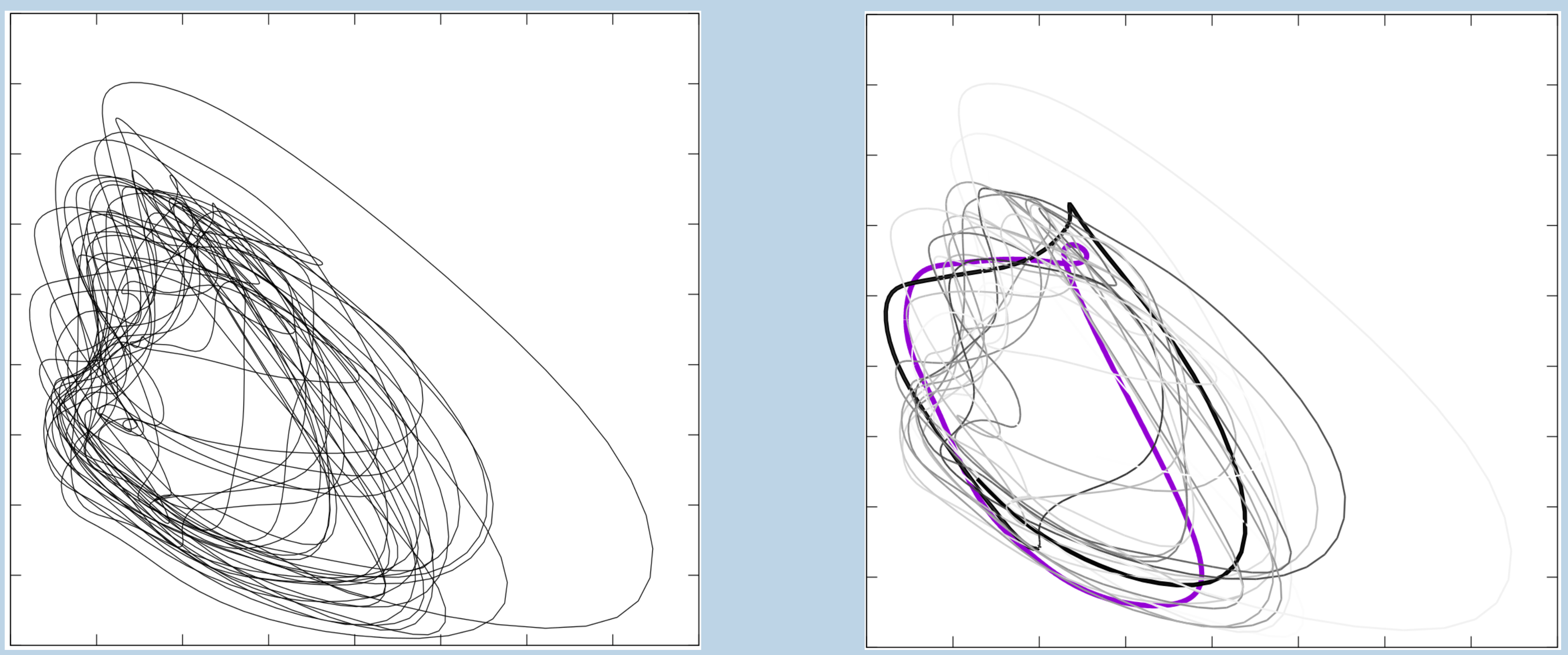

**Figure 2:** From the initial chaos on the left, a guess is formed (dark line in the right) by recurrent flow analysis and then converged to the purple unstable periodic orbit.

## Using Direction data

When the residual (1) is low enough to suggest that a periodic orbit may exist we must then converge this near recurrence to a full recurrence using the Newton-GMRES Hookstep method; a lengthy and computationally expensive process that is not always successful. To improve the success rate of the convergence we compare the direction of the solution trajectory through phase space (2) as well as the euclidean distance (1).

$$V(t) = \dot{\Omega} \approx \frac{\Omega(t) - \Omega(t+dt)}{dt}, \quad A(t,T) = \frac{\Re(V(t)\cdot\overline{V(t-T)})}{|V(t)||V(t-T)|} = \cos\theta. \qquad (2)$$

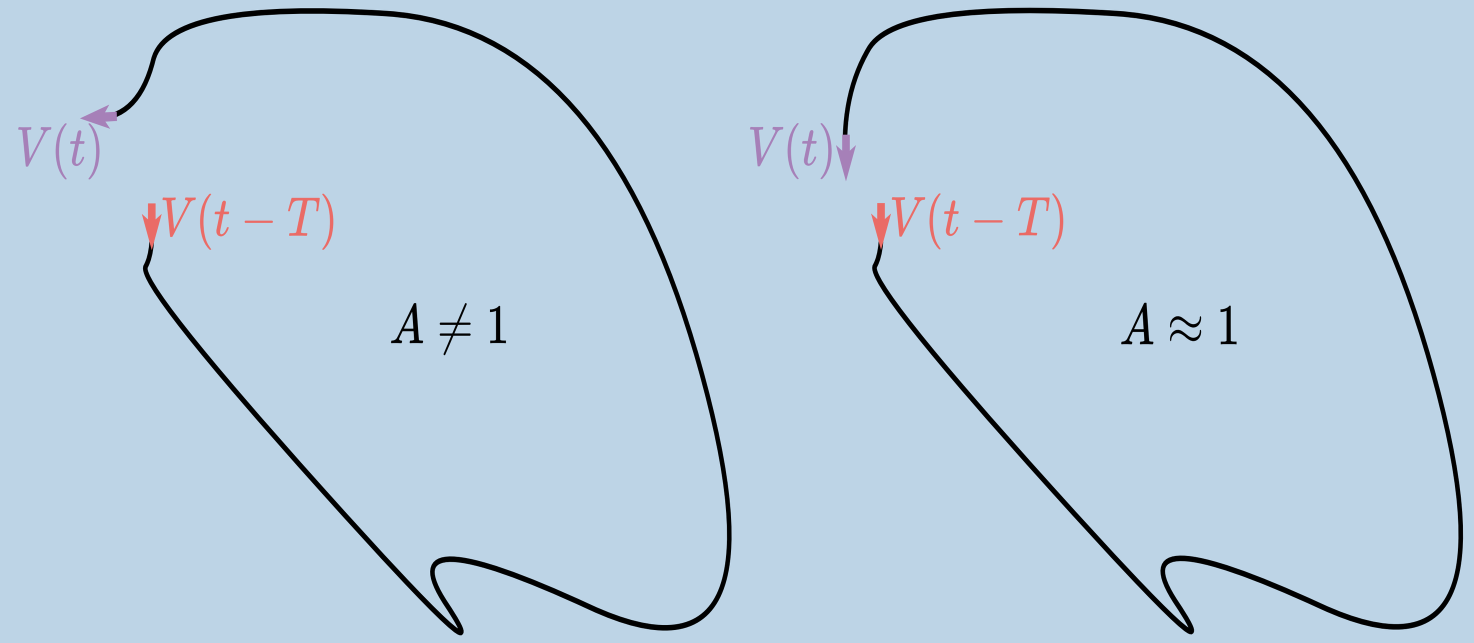


**Figure 3:** This figure indicates an near recurrent (right) trajectory that is more likely to converge than its counterpart on the left based on direction data.

## Kolmogorov flow

We create a turbulent environment in two dimensions with Kolmogorov flow, Figure 4 i.e periodic body forcing. We can observe evidence of turbulent behaviour in Figure 4, where a still of the direct numerical simulation shows the vorticity of the flow with complex structure.

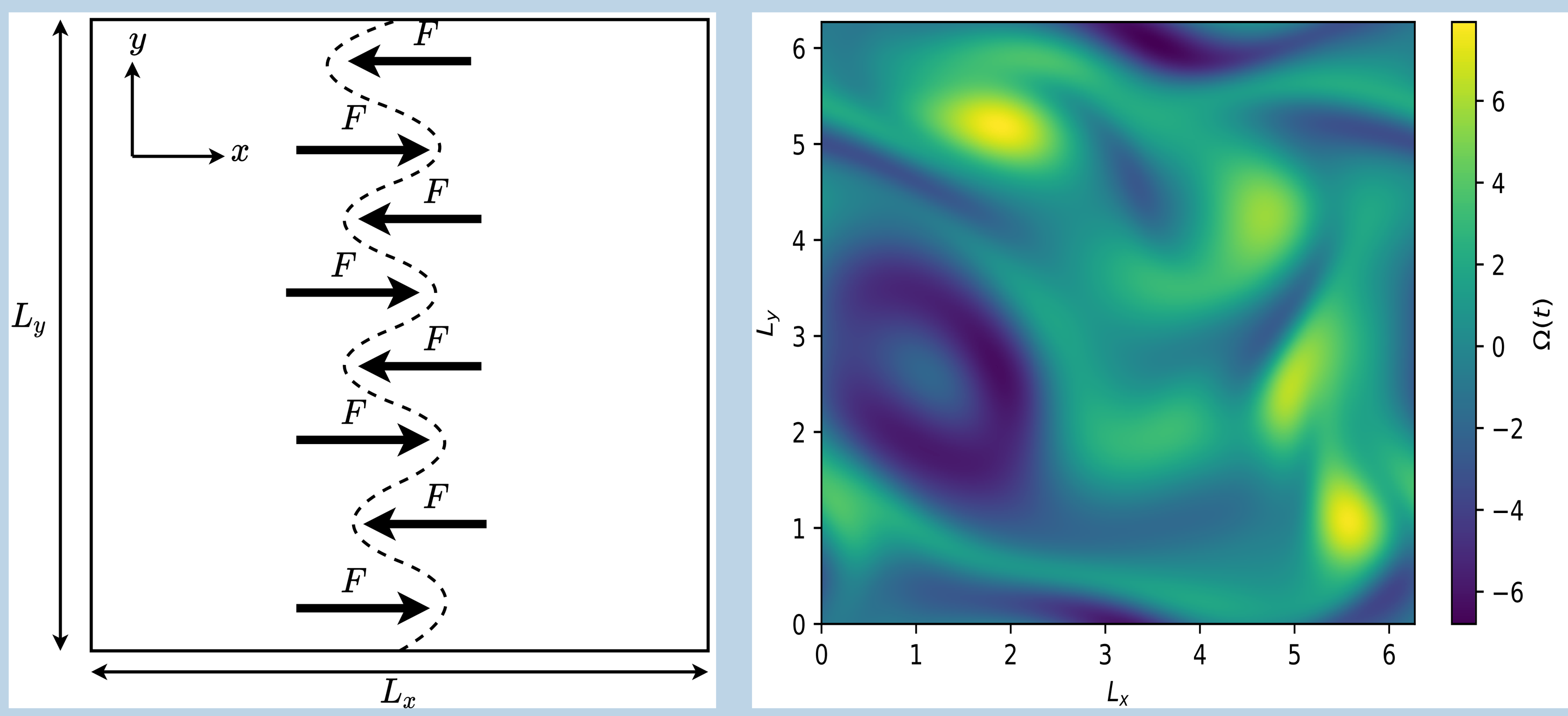


**Figure 4:** The formulation of the flow configuration of Kolmogorov flow (left). An example of a simulated Kolmogorov flow (right).

## Direction data applied to Kolmogorov flow

We verify that the new measure (1) can capture periodic behaviour by using a known unstable periodic orbit and testing the value of $A(t,T)$ at the period, in this case $-T \approx 16.8$. We see clearly that $A \to 1$ as $-T \to 16.8$, thus we can conclude that the direction data will detect periodic behaviour.

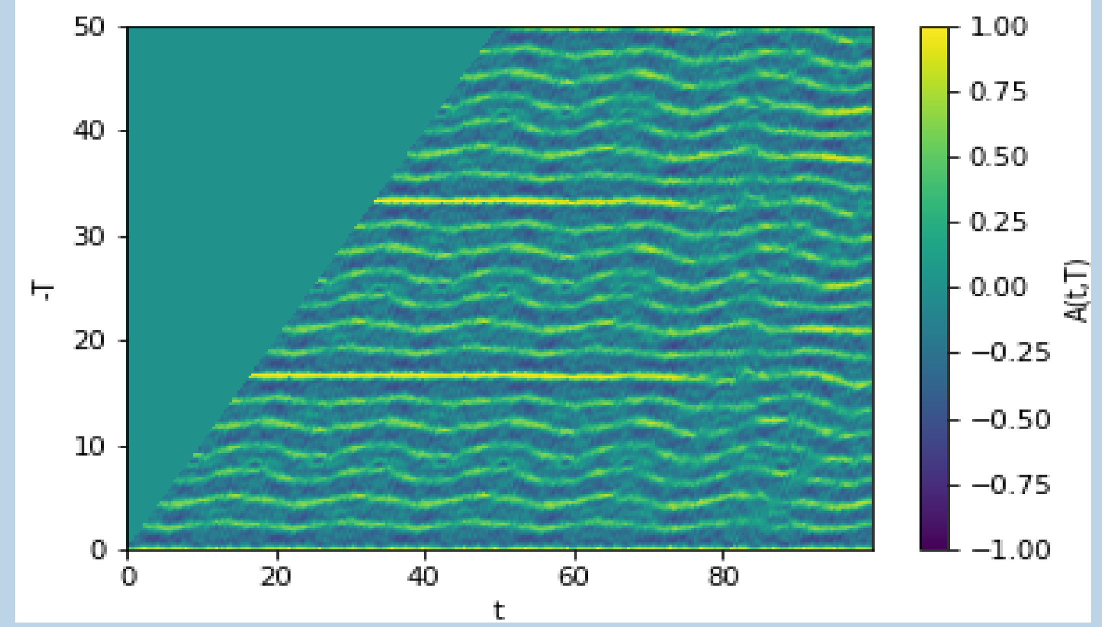


**Figure 5:** The straight yellow lines at $-T \approx 16, 32$ represent the converged periodic orbit with period 16 being detected by the direction data.

## Future work

The next step is to merge the two residuals together and apply a weighted average to find the optimum combination of measures that will improve percentage of successful applications of the Newton-GMRES Hookstep code. Figure 6 shows an example of the combined residuals plotted over 200 time units with bands of near recurrent behaviour at $10 < t < 75$.

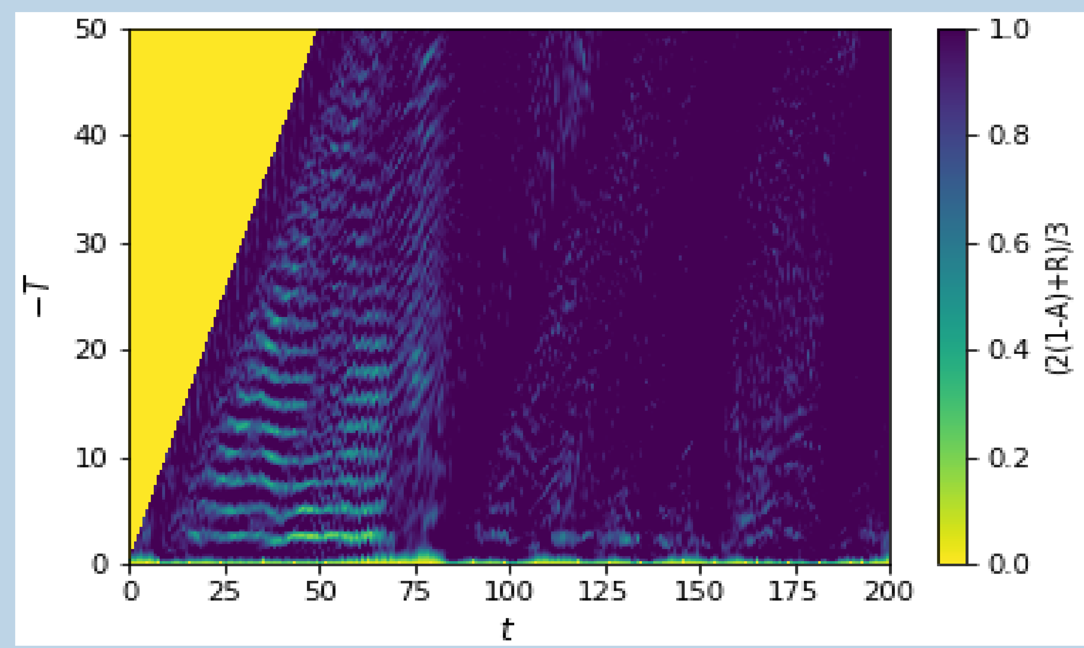


**Figure 6:** A combined two thirds residual plot for Kolmogorov flow, two thirds of the residual is derived from the direction data. Yellow bands indicate near recurrent behaviour.